

Universality and the Hagedorn Temperature

hep-th/0709xxx

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- AdS/CFT and Universality
 - Application to QCD
- The Hagedorn transition
 - Phase transitions
- Application to deformed pp-wave
- Conclusions

AdS/CFT Duality Overview

String theory
on asymptotically
 $AdS_d \times X^{10-d}$ \simeq $d - 1$ dimensional
gauge theory with
UV conformal fixed points

Classical background \simeq Ground state

Propagating string spectrum \simeq Gauge invariant states

This correspondence has the potential to be very useful, being a strong/weak duality - weak, semi-classical (**read: potentially calculable**) gravity is dual to strongly coupled gauge theory.

- **Non-perturbative effects**, previously only accessible by computationally heavy lattice calculations, may now be found via a **semi-classical GR system**.
- Strong *dynamical* effects, inaccessible to Euclidean lattice simulations, can be found.
- The potential for application to QCD at large coupling has recently received a great deal of attention.

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Asymptotic freedom allows perturbative calculations at very high energies, but...

- Deconfinement temperature still too low to reliably trust pQCD.
- Perturbation theory predicts high viscosity, ruining perfect fluid assumptions.
- Controversial recent findings at RHIC indicate extremely low viscosity, applicability of perfect fluid approximations.

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... which is precisely what we **don't** have.

Known duals

- are deformations of $\mathcal{N} = 4$ SYM (with a few exceptions)
- often involve some amount of supersymmetry
- almost exclusively deal with a large number of colors (e.g., for $SU(N)$)
- Have we failed before we've begun?

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There is hope. . .

- It has been found that all strongly coupled gauge theory plasmas with gravity duals satisfy a formula for the shear viscosity in relation to the entropy density

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \times 10^{-13} K \cdot s$$

- It is conjectured that this is a *universal bound* for finite coupling
- It is tantalizing to note that RHIC's findings are consistent with this value. . .
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The broad generality of this result has stimulated a great deal of research:

- are there other such **universal quantities**?
- can they be reliably related to QCD - **how universal are they**?
- other variables which are **measurable at RHIC**?

Examples:

- η/s (hep-th/0309213, hep-th/0311175, ...)
- ξ/η (hep-th/0703093)
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Canonical Ensemble

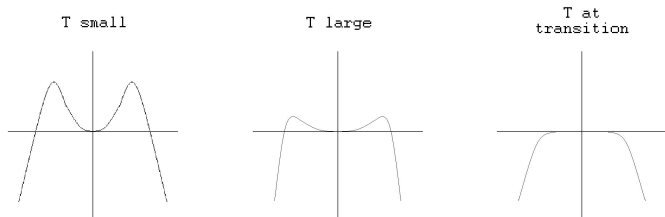
$$\begin{aligned}\mathcal{Z} = e^{-\beta F} &\equiv \text{tr } e^{-\beta H} \\ &= \int d\epsilon \rho(\epsilon) e^{-\beta \epsilon}\end{aligned}$$

In string theory, $\rho \sim e^{\beta_H \epsilon}$, so that at temperatures above $k_B T = \beta_H^{-1}$, \mathcal{Z} is not well defined.

Hagedorn Temperature

What does this imply?

- 1 Divergent free energy \Rightarrow Limiting Temperature
- 2 Finite free energy \Rightarrow Phase Transition
 - Thermal tachyon as order parameter
 - $m^2 = -4 + (1/2\pi T)^2$



- $F \sim \mathcal{O}(1) \rightarrow F \sim \mathcal{O}(g^{-2}) \sim \mathcal{O}(N^2)$
- New DOF are necessary

Hagedorn Temperature

Similar in large N gauge theory (QCD with large number of colours)

- Confined states are mesons/baryons/glueballs
 - $F \sim \mathcal{O}(1)$
- Above deconfinement temperature - free gluons
 - $F \sim \mathcal{O}(N^2)$

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Physical interest in exploring the Hagedorn transition

- Understanding strongly coupled deconfinement
- A more (?) fundamental understanding of string DOF
- The possibility of universality of T_H
e.g. Gursoy (hep-th/0602215)
- **Goal:** Understanding Hagedorn in different spacetimes

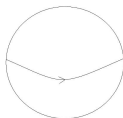
Obstructions

- Requires explicit spectrum to find DOS
- Handful of calculable spacetimes
- e.g., maximally SUSY pp-wave as a Penrose limit of $AdS_5 \times S^5$

$$ds^2 = R^2 ds_{AdS_5}^2 + R^2 d\Omega_5^2$$

↓ Penrose limit

$$ds_{pp}^2 = -2dx^+ dx^- - (\mathbf{x}^2 + \mathbf{y}^2) (dx^+)^2 + d\mathbf{x}^2 + d\mathbf{y}^2$$



Maximally SUSY pp-wave

- Transverse oscillating spectrum

$$m_n = \sqrt{1 + \frac{n^2}{(2p_-)^2}}$$

- Exponential density of states

$$\rho(\epsilon) = \exp\left(64 \sum_{\ell=1}^{\infty} \frac{K_1(\lambda\ell)}{\ell} \epsilon\right)$$

- Finite free energy \Rightarrow phase transition

$$F \sim -\sqrt{\beta - \beta_H} + \text{regular}$$

We examine the pp-wave under a SUSY breaking deformation...

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Deformed pp-wave

$$ds^2 = R^2 ds_{AdS_5}^2 + R^2 d\Omega_5^2$$

↓ SUSY-breaking $SL(2, R)$

$$ds^2 = R^2 ds_{AdS_5}^2 + R^2 (d\Omega_5^\gamma)^2$$

↙ Penrose limits ↘

$$ds_{(J,0,0)}^2$$

$$ds_{(J,J,J)}^2$$



(J, J, J) Orbit

$$\begin{aligned} \mathbf{ds}_{(J,J,J)}^2 &= -2dx^+ dx^- - \left(\mathbf{x}^2 + \frac{4\gamma^2}{3 + \gamma^2} (y_1^2 + y_2^2) \right) (dx^+)^2 \\ &+ \frac{4\sqrt{3}}{\sqrt{3 + \gamma^2}} (y_1 dy_3 + y_2 dy_4) dx^+ + d\mathbf{x}^2 + d\mathbf{y}^2 \\ \mathbf{B}_2 &= \frac{\gamma}{\sqrt{3}} dy_3 \wedge dx^+ + \frac{2\gamma}{\sqrt{3 + \gamma^2}} dx^+ \wedge (y_1 dy_4 - y_2 dy_3) \end{aligned}$$

- Spectrum identical to maximally SUSY pp-wave
- Same partition function
- **Hagedorn behavior unchanged**

(J, 0, 0) Orbit

$$\begin{aligned} ds_{(J,0,0)}^2 &= -2dx^+ dx^- - (\mathbf{x}^2 + (1 + \gamma^2)\mathbf{y}^2) (dx^+)^2 + d\mathbf{x}^2 + d\mathbf{y}^2 \\ \mathbf{B}_2 &= \gamma(y_1 dy_2 - y_2 dy_1 + y_3 dy_4 - y_4 dy_3) \wedge dx^+ \end{aligned}$$

- Degeneracy broken

$$\begin{aligned} \omega_n^x &= \sqrt{1 + \frac{n^2}{(2p_-)^2}} \\ \omega_n^{y^\pm} &= \sqrt{1 + \left(1 \pm \frac{n}{2p_-}\right)^2} \end{aligned}$$

- Non-trivial deformation of partition function and high energy $\rho(\epsilon)$
- **Hagedorn behavior unchanged**

Comparing to gauge theory

- In the gauge theory, the Penrose limit is dual to the BMN limit - examining gauge invariant operators with large R-charge

$$\psi = \text{Tr } \phi_{i_1} \phi_{i_2} \dots \phi_{i_J}$$

- Can be identified with a spin chain of length J
- Harnack & Orselli (hep-th/0608115) found an interesting limit in the undeformed case which allowed computation of Hagedorn temperature
- Next step is to understand Hagedorn in the deformed spin chain

Conclusions

- Universal thermodynamic behavior may shed light on gauge/gravity duality
- Hagedorn transition related to QCD deconfinement
- One can take two physically distinct Penrose limits of deformed $\text{AdS}_5 \times S^5_\gamma$
- In both deformed pp-waves, the Hagedorn behavior is unaffected
- There exists some form of universality which remains to be explored